

# Prediction of positive parity $B_s$ mesons and search for the $X(5568)$

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We use a combination of quark-antiquark and  $B^{(*)}K$  interpolating fields to predict the mass of two QCD bound states below the  $B^*K$  threshold in the quantum channels  $J^P = 0^+$  and  $1^+$ . The mesons correspond to the b-quark cousins of the  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  and have not yet been observed in experiment, even though they are expected to be found by LHCb. In addition to these predictions, we obtain excellent agreement of the remaining p-wave energy levels with the known  $B_{s1}(5830)$  and  $B_{s2}^*(5840)$  mesons. The results from our first principles calculation are compared to previous model-based estimates. More recently the D0 collaboration claimed the existence of an exotic resonance  $X(5568)$  with exotic flavor content  $\bar{b}s\bar{d}u$ . If such a state with  $J^P = 0^+$  exists, only the decay into  $B_s\pi$  is open which makes a lattice search for this state much cleaner and simpler than for other exotic candidates involving heavy quarks. We conclude, however, that we do not find such a candidate in agreement with a recent LHCb result.

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In these proceedings we summarize two recently published lattice QCD studies [1, 2] of states close to multi-particle thresholds.

## 1. Prediction of positive parity $B_s$ mesons

The discovery of the  $D_{s0}^*(2317)$  by BaBar [3] and the subsequent discovery of the  $D_{s1}(2460)$  more than 10 years ago revealed an unexpected peculiarity: unlike expected by potential models, these states turned out to be narrow states below the  $DK$  and  $D^*K$  thresholds. Moreover their mass is roughly equal to the mass of their non-strange cousins, which immediately sparked speculations about their structure in terms of quark content, with popular options including both tetraquark and molecular structures.

The corresponding  $J^P = 0^+$  and  $1^+$  states in the spectrum of  $B_s$  hadrons have not been established in experiment. Given the success of recent lattice QCD calculations of the  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$  [4, 5], it is therefore interesting to see if a prediction of these positive parity  $B_s$  states from lattice QCD is feasible.

$N_L^3 \times N_T$	$N_f$	$a[\text{fm}]$	$L[\text{fm}]$	#configs	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$
$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

**Table 1:** Gauge configurations used for the simulations in these proceedings.

### 1.1 Lattice techniques

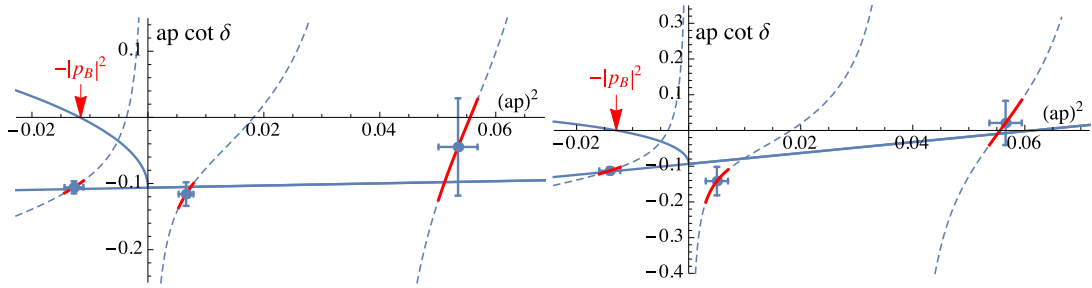
For this study we use the 2+1 flavor gauge configurations with Wilson-Clover quarks generated by the PACS-CS collaboration [6]. Table 1 shows details of the ensemble used in our simulation. Our quark sources are smeared with a Gaussian-like envelope as produced by use of the stochastic distillation technique [7]. For the heavy b-quarks in the Fermilab interpretation [8], we tune the heavy-quark hopping parameter  $\kappa_b$  for the spin averaged kinetic mass  $M_{\overline{B}_s} = (M_{B_s} + 3M_{B_s^*})/4$  to assume its physical value. The energy splittings we determine are expected to be close to physical in this setup. For technical details on the tuning of the heavy-quark hopping parameter please refer to [5, 1]. We work with a partially quenched strange quark and used the  $\phi$  meson and  $\eta_s$  to set the strange quark mass, obtaining  $\kappa_s = 0.13666$  [5]. Table 2 shows examples of mass splittings extracted with this setup. Notice that the uncertainties provided in this table are statistical and scale-setting uncertainties only. Nevertheless the agreement with experiment is mostly excellent, indicating that the remaining discretization effects are small.

For the construction of the correlation matrix used to extract the finite volume energies, our study takes into account both quark-antiquark as well as B-K structures. The basis is similar to our study of the  $D_s$  spectrum [4, 5], where this approach allowed us to obtain reliable energy levels for the  $D_{s0}^*(2317)$  and  $D_{s1}(2460)$ . For elastic s-wave scattering the Lüscher relation [9] relating the finite volume spectrum to the phase shift  $\delta$  of the infinite volume scattering amplitude is given by

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi}L} Z_{00}(1; q^2) \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2. \quad (1.1)$$

	Lattice [MeV]	Exp. [MeV]
$m_{B^*} - m_B$	46.8(7.0)(0.7)	45.78(35)
$m_{B_{s^*}} - m_{B_s}$	47.1(1.5)(0.7)	$48.7^{+2.3}_{-2.1}$
$m_{B_s} - m_B$	81.5(4.1)(1.2)	87.35(23)
$m_Y - m_{\eta_b}$	44.2(0.3)(0.6)	62.3(3.2)
$2m_{\bar{B}} - m_{\bar{b}b}$	1190(11)(17)	1182.7(1.0)
$2m_{\bar{B}_s} - m_{\bar{b}b}$	1353(2)(19)	1361.7(3.4)
$2m_{B_c} - m_{\eta_b} - m_{\eta_c}$	169.4(0.4)(2.4)	167.3(4.9)

**Table 2:** Selected mass splittings (in MeV) of mesons involving bottom quarks compared to the values from the PDG [11]. A bar denotes spin average. Errors are statistical and scale-setting only.



**Figure 1:** Plots of  $ap \cot \delta(p)$  vs.  $(ap)^2$  for  $B^{(*)}K$  scattering in  $s$ -wave. Circles are values from our simulation; red lines indicate the error band following the Lüscher curves (broken lines). The full line gives the linear fit to the points. Below threshold  $|p|$  is added and the zero of the combination indicates the bound state position in infinite volume. Displayed uncertainties are statistical only.

We perform an effective range approximation with the  $s$ -wave scattering length  $a_0$  and effective range  $r_0$ . The resulting parameters and the mass of the resulting binding momentum (from  $\cot(\delta(p)) = i$ ) are shown in Figure 1. We obtain

$$\begin{aligned}
 a_0^{BK} &= -0.85(10) \text{ fm} & a_0^{B^*K} &= -0.97(16) \text{ fm} \\
 r_0^{BK} &= 0.03(15) \text{ fm} & r_0^{B^*K} &= 0.28(15) \text{ fm} \\
 M_{B_{s0}^*} &= 5.711(13) \text{ GeV} & M_{B_{s1}^*} &= 5.750(17) \text{ GeV}
 \end{aligned} \tag{1.2}$$

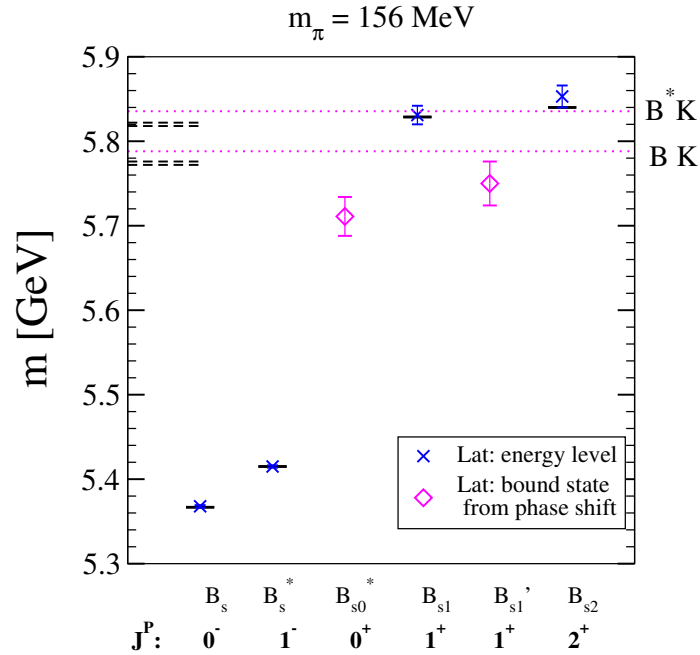
where the uncertainty on the bound state mass is statistical only. A full uncertainty estimate is given in Table 3 and explained in more detail in [1].

## 1.2 Resulting prediction of positive parity $B_s$ mesons

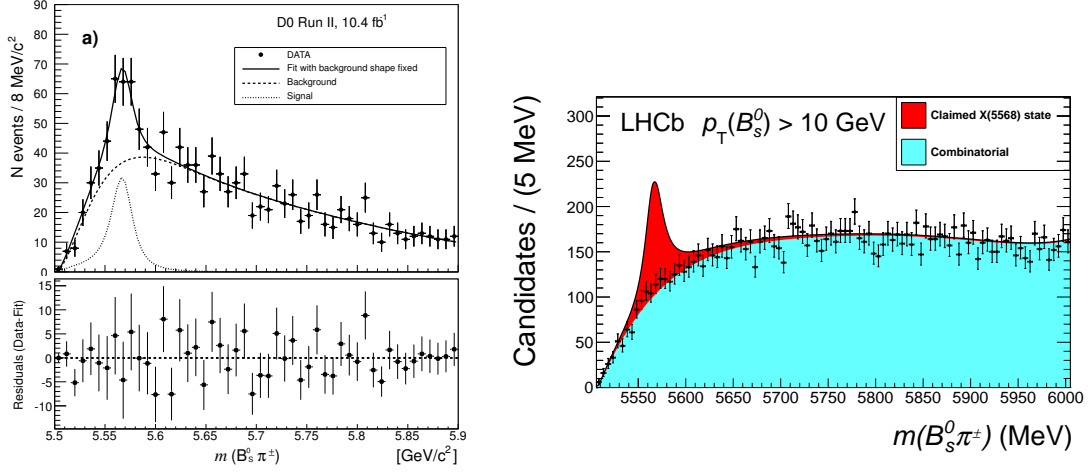
Figure 2 shows our final results for the spectrum of  $s$ -wave and  $p$ -wave  $B_s$  states. For values of masses in MeV we quote  $M = \Delta M^{\text{lat}} + M_{B_s}^{\text{exp}}$  where we substitute the experimental  $B_s$  spin average in accordance with our tuning. The states with blue symbols result from a naive determination of the finite volume energy levels (statistical uncertainty only). Notice that the  $j = \frac{3}{2}$  states agree well with the experimental  $B_{s1}(5830)$  and  $B_{s2}^*(5840)$  as determined by CDF/D0 and LHCb [11]. The  $B_s$  states with magenta symbols indicate the bound state positions extracted using Lüscher's method

source of uncertainty	expected size [MeV]
heavy-quark discretization	12
finite volume effects	8
unphysical Kaon, isospin & EM	11
b-quark tuning	3
dispersion relation	2
spin-average (experiment)	2
scale uncertainty	1
3 pt vs. 2 pt linear fit	2
total (added in quadrature)	19

**Table 3:** Systematic uncertainties in the mass determination of the below-threshold states with quantum numbers  $J^P = 0^+, 1^+$ . The heavy-quark discretization effects are quantified by calculating the Fermilab-method mass mismatches and employing HQET power counting [10] with  $\Lambda = 700$  MeV. The finite volume uncertainties are estimated conservatively by the difference of the lowest energy level and the complex pole position. The last line gives the effect of using only the two points near threshold for the effective range fit. The total uncertainty has been obtained by adding the single contributions in quadrature.



**Figure 2:** Spectrum of s-wave and p-wave  $B_s$  states from our simulation. The blue states are naive energy levels, while the bound state energy of the states in magenta results from an effective range approximation of the phase shift data close to threshold. The black lines are the energy levels from the PDG [11]. The error bars on the blue states are statistical only, while the errors on the magenta states show the full (statistical plus systematic) uncertainties.



**Figure 3:** Left pane:  $B_s^0 \pi^\pm$  invariant mass distribution from D0 [12] (after applying a cone cut). Right pane:  $B_s^0 \pi^\pm$  invariant mass distribution by LHCb [13] shown in black symbols with a signal component corresponding to  $\rho_X = 8.6\%$  as observed by D0 shown in red.

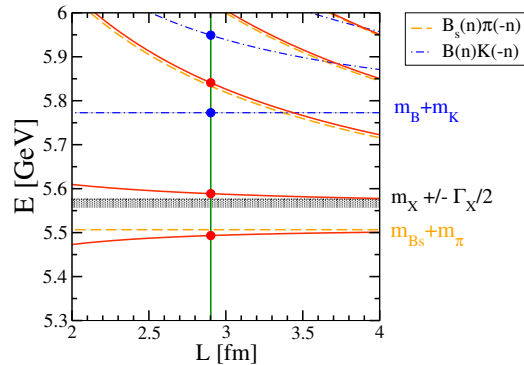
and taking into account the sources of uncertainty detailed in Table 3. Notice that our Lattice QCD calculations yields bound states well below the  $B^{(*)}K$  thresholds.

## 2. $B_s \pi^+$ scattering and search for the $X(5568)$

Recently, the D0 collaboration has reported evidence for a peak in the  $B_s \pi^+$  invariant mass not far above threshold [12]. This peak is attributed to a resonance dubbed  $X(5568)$  with the resonance mass  $m_X$  and width  $\Gamma_X$ ,

$$\begin{aligned} m_X &= 5567.8 \pm 2.9^{+0.9}_{-1.9} \text{ MeV} , \\ \Gamma_X &= 21.9 \pm 6.4^{+5.0}_{-2.5} \text{ MeV} . \end{aligned} \quad (2.1)$$

Decay of this resonance into  $B_s \pi^+$  implies an exotic flavor structure with the minimal quark content  $\bar{b}s\bar{d}u$ . Most model studies which accommodate a  $X(5568)$  propose spin-parity quantum numbers  $J^P = 0^+$ . Short after D0 reported their results, the LHCb collaboration investigated the cross-section as a function of the  $B_s \pi^+$  invariant mass with increased statistics and did not find any peak in the same region [13]. Figure 3 shows both the plot from D0 (left pane) and the data from LHCb (right pane), where the red shaded region illustrates the signal expectation given the ratio of yields  $\rho_X$  determined by D0.



**Figure 4:** Analytic predictions for energies  $E(L)$  of eigenstates as a function of lattice size  $L$ .

## 2.1 Expected signature for a resonance in $B_s\pi^+$

The presence of an elastic resonance with the parameters of the  $X(5568)$  would lead to a characteristic pattern of finite volume energy levels corresponding to QCD eigenstates with given quantum numbers for finite spatial size  $L$ .

Figure 4 shows analytic predictions for energies of eigenstates for an elastic resonance in  $B_s\pi$  (with  $J^P = 0^+$ ) as a function of the lattice size  $L$  as determined from Lüscher's formalism [9]. Red solid lines are  $B_s\pi$  eigenstates in the scenario with resonance  $X(5568)$ ; orange dashed lines are  $B_s\pi$  eigenstates when  $B_s$  and  $\pi$  do not interact; blue dot-dashed lines are  $B^+\bar{K}^0$  eigenstates when  $B^+$  and  $\bar{K}^0$  do not interact; the grey band indicates the position of  $X(5568)$  from the D0 experiment [12]. The lattice size  $L = 2.9$  fm, used in our simulation, is marked by the vertical line. Note that the resonant scenario predicts an eigenstate near  $E \simeq m_X$  (red solid), while there is no such eigenstate for  $L = 2 - 4$  fm in a scenario with no or small interaction between  $B_s$  and  $\pi^+$  (orange dashed). In the unlikely scenario of a deeply bound  $BK$  state, the simulation would result in an eigenstate with  $E \approx m_X$  up to exponentially small corrections in  $L$ .

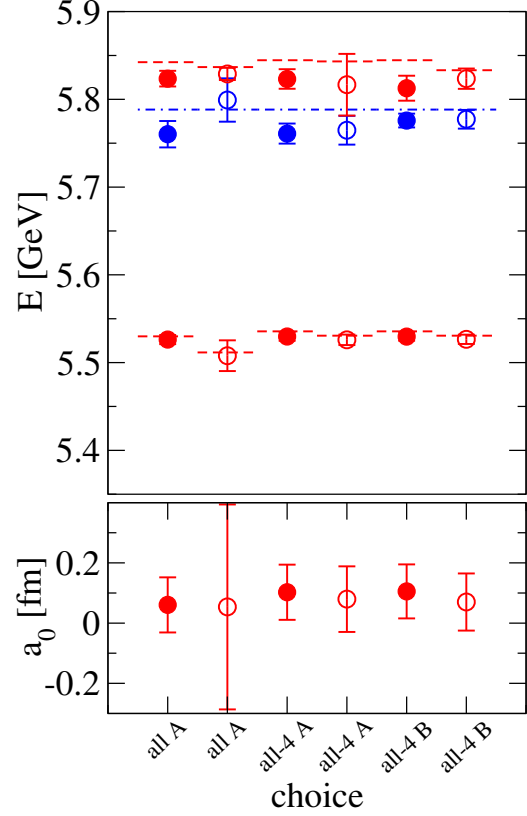
## 2.2 Simulation details

In our simulation we use the PACS-CS ensemble [6] from Table 1. The interpolator basis

$$\begin{aligned} O_{1,2}^{B_s(0)\pi(0)} &= [\bar{b}\Gamma_{1,2}s] (\mathbf{p}=0) [d\bar{\Gamma}_{1,2}u] (\mathbf{p}=0), \\ O_{1,2}^{B_s(1)\pi(-1)} &= \sum_{\mathbf{p}=\pm\mathbf{e}_{x,y,z}} [\bar{b}\Gamma_{1,2}s] (\mathbf{p}) [d\bar{\Gamma}_{1,2}u] (-\mathbf{p}), \\ O_{1,2}^{B(0)K(0)} &= [\bar{b}\Gamma_{1,2}u] (\mathbf{p}=0) [d\bar{\Gamma}_{1,2}s] (\mathbf{p}=0), \end{aligned}$$

consisting of both  $B_s\pi$  and  $BK$  interpolators, is employed.

Figure 5 shows the eigenstates determined from our simulation for various choices. The sets with full symbols are from correlated fits while open symbols result from uncorrelated fits. Notation “all” refers to the full set of gauge configurations while “all-4” refers to the set with four (close to exceptional) gauge configurations removed. Set A is from interpolator basis  $O_1^{B_s(0)\pi(0)}$ ,  $O_1^{B_s(1)\pi(-1)}$ ,  $O_1^{B(0)K(0)}$  while set B results from a larger basis  $O_1^{B_s(0)\pi(0)}$ ,  $O_{1,2}^{B_s(1)\pi(-1)}$ ,  $O_{1,2}^{B(0)K(0)}$ . All choices consistently result in a small scattering length  $a_0$  consistent with 0 within error.



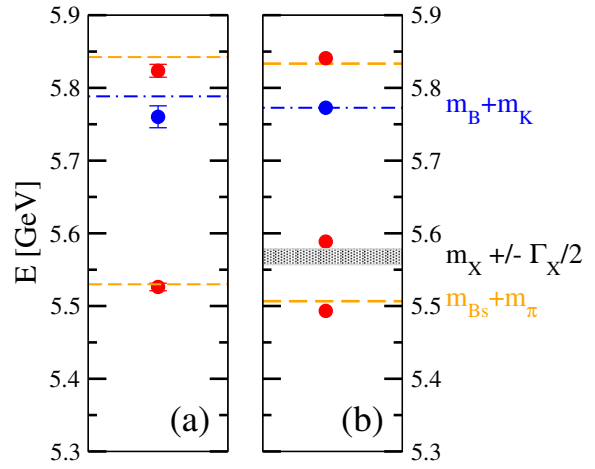
**Figure 5:** The eigenenergies of the  $\bar{b}s\bar{d}u$  system with  $J^P = 0^+$  from a lattice simulation for the choices detailed in the text.

### 2.3 Conclusions from comparing analytic predictions and lattice energy levels

Figure 6 shows the eigenenergies of the  $\bar{b}s\bar{d}u$  system with  $J^P = 0^+$  calculated on the lattice (left pane) compared to the analytic prediction based on the  $X(5568)$  as observed by D0 (right pane). Unlike expected for the case of a resonance with the parameters of the  $X(5568)$ , our lattice simulation at close-to-physical quark masses does not yield a second low-lying energy level. Our results therefore do not support the existence of  $X(5568)$  with  $J^P = 0^+$ . Instead, the results appear closer to the limit where  $B_s$  and  $\pi$  do not interact significantly, leading to a  $B_s\pi$  scattering length compatible with 0 within errors.

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**Figure 6:** (a) The eigenenergies of the  $\bar{b}s\bar{d}u$  system with  $J^P = 0^+$  from our lattice simulation and (b) an analytic prediction based on  $X(5568)$ , both at lattice size  $L = 2.9$  fm. The horizontal lines show energies of eigenstates  $B_s(0)\pi^+(0)$ ,  $B^+(0)\bar{K}^0(0)$  and  $B_s(1)\pi^+(-1)$  in absence of interactions; momenta in units of  $2\pi/L$  are given in parenthesis. The pane (a) shows the energies  $E = E_n^{lat} - E_{B_s}^{lat} + E_{B_s}^{exp}$  with the spin-averaged  $B_s$  ground state set to its experimental value. The pane (b) is based on the experimental mass of the  $X(5568)$  [12], given by the grey band, and experimental masses of other particles.